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Surface waves on a superconductor: beyond the weak-coupling approximation

Marina V Glumova[1], **Valeri Z Lozovski**[2] **and Dmitry V Reznik**[2]

[1] Department of Physics, National Tavrida University, 28 Yaltinska Street, 95330 Simferopol, Crimea, Ukraine
[2] Institute of Semiconductor Physics, National Academy of Sciences of Ukraine, 45 Prospekt Nauki, 03028 Kyiv, Ukraine

E-mail: reznik@mol.semicond.kiev.ua

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Abstract
A theory of electromagnetic surface wave propagation on a plane superconductor–vacuum interface is presented for a superconductor with strong coupling. The solution of a self-consistent equation for the energy of elementary excitations is obtained in terms of modified u–v transformation method. The linear response (conductivity) tensor is analysed in the framework of a non-local description allowing for the finite dimension of the Cooper pair. Within the framework of the semi-classical infinite-barrier model the surface wave dispersion relations are calculated analytically and numerically for the cases of strong-, intermediate-, and weak-coupling superconductors.

1. Introduction

Interest in studying surfaces and low-dimensional systems has heightened in the past few years [1]. The study of optical and electrical properties of a surface of solids has a fundamental and practical importance. In particular, there are problems concerned with integral optics, microelectronics and nanoelectronics, and laser technology. In this connection one should be aware of the studies of different surface effects in superconductors [2–4]. We wish to emphasize that studies of the influences of surfaces on superconductivity have been proceeding vigorously since the discovery of superhigh-temperature superconductivity. We would like especially to draw attention to some new aspects of these studies. There have been studies of the influence of surfaces on T_C by the method of molecular luminescent markers [5] and also of the influence of an external electrostatic field on the properties of HTSC (see, for example, [6] and [7]).

The study of surface electromagnetic excitations or surface electromagnetic waves (SEW) plays the leading role in investigation of surface properties. The presence on the propagation plane of SEW peaks of appreciable amplitude makes it possible to use SEW spectroscopy

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as a high-information probe for low-dimensional structures. Scanning using SEW may give information about the structure of the near field and the presence of defects on a surface. Studying the behaviour of SEW on superconductors has made it possible to investigate new types of input circuit for superhigh-frequency devices with maximally low noise levels, which are especially important in the design of quantum amplifiers with ballistic transistors.

SEW propagation on metals, semiconductors, and dielectrics has been studied in a number of works (see, for example, [8–10]). One of the main characteristics of a SEW is its dispersion relation. The dispersion relations obtained in a non-local approach for SEW on surfaces of normal metals were discussed in [11–13]. The propagation of SEW along a plane superconductor–vacuum interface was considered in [14–16], taking into account the effects of non-locality. These works were carried out in the framework of BCS theory. It is known that BCS theory [17] cannot be applied to superconductors with strong coupling. This means that the results obtained in [14–16] are limited in application to superconductors with weak coupling (generally speaking, this is a rather wide range of materials). To consider the case of strong coupling one needs to use something like Eliashberg theory [18]. Although consecutive solution of Eliashberg equations [18, 19] is rather difficult, there have been several attempts to obtain partial solutions of these equations which have proved successful (see, for example, [20–24]). There is another way to obtain the ground state of a strong-coupling superconductor, as was discussed in [25]. This way is based on the possibility of modification of the Bogoliubov u–v transformation method. Some steps for obtaining the Hamiltonian of the effective electron–electron interaction in this method were proposed in [25]. In the first stage the exact expression for the energy dependence for new single-particle excitation operators was constructed by introducing all single-particle excitation energy. In the second step the Fermi operator for the electrons was replaced by a new operator by means of the reverse Bogoliubov u–v transformation and a new Hamiltonian was obtained in terms of single-fermion excitation operators. Then, self-consistent equations for the energy spectrum of the elementary excitations were obtained. Unfortunately, to our knowledge nobody has attempted to solve these self-consistent equations. In this work, we will present the results of calculations of non-local dispersion relation for SEW on a plane superconductor–vacuum interface performed in the framework of the u–v transformations method for strong-coupling superconductors proposed in [25].

2. The non-local linear response tensor

In the work [14], Keller has developed a method for calculating the dispersion of surface waves on a weak-coupling superconductor in the framework of u–v Bogoliubov transformations. In the present work we will follow the approach developed in [14]. It seems to be convenient to use the method based on u–v transformations for considering strong-coupling superconductors.

We will suppose that excitation of the surface wave does not lead to reconstruction of the energy spectrum of the elementary excitations of a superconductor. To calculate the electrodynamic properties of a superconductor we need to study the conductivity tensor. In the random-phase approximation this tensor is given by [26]

$$\overset{\leftrightarrow}{\sigma}(\vec{q},\omega)=\frac{\mathrm{i}ne^2}{m\omega}\overset{\leftrightarrow}{U}-\frac{\mathrm{i}}{\omega}\left(\frac{e\hbar}{2m}\right)^2\frac{1}{V}\sum_{\vec{k}}(2\vec{k}+\vec{q})\otimes(2\vec{k}+\vec{q})\Big\{(u_{\vec{k}}u_{\vec{k}+\vec{q}}+v_{\vec{k}}v_{\vec{k}+\vec{q}})^2$$
$$\times[f(\varepsilon_{\vec{k}})-f(\varepsilon_{\vec{k}+\vec{q}})]\left(\frac{1}{\varepsilon_{\vec{k}+\vec{q}}-\varepsilon_{\vec{k}}+\hbar\omega}+\frac{1}{\varepsilon_{\vec{k}+\vec{q}}-\varepsilon_{\vec{k}}-\hbar\omega}\right)$$

$$+ (v_{\vec{k}}u_{\vec{k}+\vec{q}} - u_{\vec{k}}v_{\vec{k}+\vec{q}})^2[1 - f(\varepsilon_{\vec{k}}) - f(\varepsilon_{\vec{k}+\vec{q}})] \times \left(\frac{1}{\varepsilon_{\vec{k}+\vec{q}} + \varepsilon_{\vec{k}} + \hbar\omega} + \frac{1}{\varepsilon_{\vec{k}+\vec{q}} + \varepsilon_{\vec{k}} - \hbar\omega}\right)\Bigg\}. \tag{1}$$

Here $\varepsilon_{\vec{k}}$ is the energy of quasi-particle excitations,

$$f(\varepsilon_{\vec{k}}) = \left[\exp\left(\frac{\varepsilon_{\vec{k}}}{k_B T}\right) + 1\right]^{-1} \tag{2}$$

is the Fermi–Dirac distribution function, and

$$u_{\vec{k}} = \frac{1}{\sqrt{2}}\left(1 + \frac{\tilde{\mathcal{E}}^*_{\vec{k}}}{\sqrt{\tilde{\mathcal{E}}^{*2}_{\vec{k}} + \tilde{\Delta}^2_{\vec{k}}}}\right)^{1/2}, \tag{3}$$

$$v_{\vec{k}} = \frac{1}{\sqrt{2}}\left(1 - \frac{\tilde{\mathcal{E}}^*_{\vec{k}}}{\sqrt{\tilde{\mathcal{E}}^{*2}_{\vec{k}} + \tilde{\Delta}^2_{\vec{k}}}}\right)^{1/2} \tag{4}$$

are the coefficients of the Bogoliubov transformation, which express the probability amplitudes for the pair state $(\vec{k}\uparrow, -\vec{k}\downarrow)$ being empty and occupied, respectively; $\tilde{\mathcal{E}}^*_{\vec{k}}$ and $\tilde{\Delta}_{\vec{k}}$ are the effective free-electron energy and energy gap parameter, respectively.

As shown in [14], in a coordinate system in which the wavevector $\vec{q}$ is directed along a certain axis (let it for definiteness be the OZ-axis), the conductivity tensor $\overleftrightarrow{\sigma}(\omega, \vec{q})$ takes the diagonal form

$$\overleftrightarrow{\sigma}(\omega, \vec{q}) = \begin{bmatrix} \sigma_T(\vec{q}, \omega) & 0 & 0 \\ 0 & \sigma_T(\vec{q}, \omega) & 0 \\ 0 & 0 & \sigma_L(\vec{q}, \omega) \end{bmatrix}. \tag{5}$$

$\sigma_T(\vec{q}, \omega)$ and $\sigma_L(\vec{q}, \omega)$ are the transverse and longitudinal parts of the response tensor. Some simplifications can be made. First, as usual, the summation in equation (1) can be replaced by integration according to standard rules [27]. Second, for $q \ll k$ ($k \approx k_F$, the Fermi momentum) it is convenient to perform a Taylor expansion of $\overleftrightarrow{\sigma}(\omega, \vec{q})$ around $q = 0$. Thus, the expression for $\overleftrightarrow{\sigma}(\omega, \vec{q})$, to first order in q^2 can be written as

$$\overleftrightarrow{\sigma}(\vec{q}, \omega) \approx \overleftrightarrow{\sigma}(0, \omega) + \frac{1}{2}q^2\frac{\partial^2\overleftrightarrow{\sigma}(0, \omega)}{\partial q^2}. \tag{6}$$

As noted in [14], in this case $\overleftrightarrow{\sigma}(\vec{q}, \omega)$ may be written in the following form (here $\vec{q} = (0, 0, q)$, $\vec{e}_q = \vec{q}/q = (0, 0, 1)$):

$$\overleftrightarrow{\sigma}^{nl}(\vec{q}, \omega) = \left(\frac{ine^2}{m\omega}\right)\left[\overleftrightarrow{U} + \alpha(\omega, T)\left(\frac{q}{\omega}\right)^2(\overleftrightarrow{U} + 2\vec{e}_q \otimes \vec{e}_q)\right], \tag{7}$$

where $\alpha(\omega, T)$ is the linear response function. Then σ_T and σ_L may be written as

$$\sigma_T(\omega, \vec{q}) = \left(\frac{ine^2}{m\omega}\right)\left[1 + \alpha(\omega, T)\left(\frac{q}{\omega}\right)^2\right], \tag{8}$$

$$\sigma_L(\omega, \vec{q}) = \left(\frac{ine^2}{m\omega}\right)\left[1 + 3\alpha(\omega, T)\left(\frac{q}{\omega}\right)^2\right]. \tag{9}$$

In the calculation of the linear response one needs to consider the dispersion of quasi-particle excitation in the superconductor. Studies of waves localized on a plane BCS superconductor–vacuum interface were carried out in [14–16]. The approach taking into account the effects of non-locality of electrodynamical interactions developed in [28, 29] was used to solve this problem in [14–16]. Our work discusses the dispersion relations for SEW in the case of strong-coupling superconductors in the spirit of the works [14] and [25].

The modified method of u–v transformation allows us to obtain the ground superconducting state directly from the standard electron–phonon Hamiltonian (see, for example, the textbook [30]) without making any assumptions as regards the value of the electron–phonon coupling constant. In the framework of the u–v transformation method,

$$a_{\vec{k},\sigma} = u_{\vec{k}}\alpha_{\vec{k},\sigma} + \sigma v_{\vec{k}}\alpha^{+}_{-\vec{k},-\sigma}, \qquad a^{+}_{\vec{k},\sigma} = u_{\vec{k}}\alpha^{+}_{\vec{k},\sigma} + \sigma v_{\vec{k}}\alpha_{-\vec{k},-\sigma} \tag{10}$$

are directly inserted into the electron–phonon Hamiltonian. Then, using evaluation equations for the creation and annihilation operators for phonons, the phonon operators are excluded from the Hamiltonian. As a result, the superconducting state is described by α-operators given by equations (10). The essential feature of this procedure is the fact that the new (α) operators are operators for single-fermion excitations of the *superconducting* phase. The time dependences of these operators are explicitly given by the equations

$$\alpha^{+}_{\vec{k},\sigma} = \tilde{\alpha}^{+}_{\vec{k},\sigma}\mathrm{e}^{\mathrm{i}\varepsilon_{\vec{k}}t}, \qquad \alpha_{\vec{k},\sigma} = \tilde{\alpha}_{\vec{k},\sigma}\mathrm{e}^{-\mathrm{i}\varepsilon_{\vec{k}}t}, \tag{11}$$

with $\varepsilon_{\vec{k}}$ the energy of a single-fermion excitation of the superconductor state. It needs to be emphasized that the energy of the fermion excitations in the superconducting state is directly introduced as we include the time dependence of the creation and annihilation operators (equations (11)) for *fermion excitation in the superconducting state*. Since the operators (equations (11)) are the ones for elementary excitations of a superconducting state, diagonalization of the electron–phonon Hamiltonian in terms of $\alpha^{+}_{\vec{k},\sigma}\alpha_{\vec{k},\alpha}$ operators gives in the general case [25]

$$H_S = H_S^0 + H_S^2, \tag{12}$$

with

$$H_S^0 = \sum_{\vec{k},\sigma} F(\vec{k}, \varepsilon_{\vec{k}})\alpha^{+}_{\vec{k},\sigma}\alpha_{\vec{k},\sigma} + \sum_{\vec{k},\sigma} Q(\vec{k}, \varepsilon_{\vec{k}})(\alpha^{+}_{\vec{k},\sigma}\alpha^{+}_{-\vec{k},-\sigma} + \mathrm{H.c.}) \tag{13}$$

the quadratic part, and H_S^2 that part of the Hamiltonian describing scattering of quasi-particles by quasi-particles. The parameters $F(\vec{k}, \varepsilon_{\vec{k}})$ and $Q(\vec{k}, \varepsilon_{\vec{k}})$ are complicated functions of the wavevector and energy. Their explicit forms are given in [25]. Because the ground state of the quasi-particle system (in the superconductor state) is described by the Hamiltonian (equation (12)), one obtains the following equations of self-consistency:

$$F(\vec{k}, \varepsilon_{\vec{k}}) = \varepsilon_{\vec{k}} \tag{14}$$

and

$$Q(\vec{k}, \varepsilon_{\vec{k}}) = 0. \tag{15}$$

These equations can be rewritten in terms of the energy spectrum and gap parameter. The energy gap parameter and reduced single-electron energy satisfy a system of self-consistency equations:

$$\Delta_{\vec{k}} = -\frac{V}{(2\pi)^3}\int \mathrm{d}\vec{q}\,\mathcal{R}^{(-)}_{\vec{k},\vec{k}'}\frac{\Delta_{\vec{k}'}}{\sqrt{\mathcal{E}^{*2}_{\vec{k}'} + \Delta^2_{\vec{k}'}}}, \tag{16}$$

$$\mathcal{E}^{*}_{\vec{k}} = \mathcal{E}_{\vec{k}} + \frac{V}{(2\pi)^3}\int \mathrm{d}\vec{q}\,\mathcal{R}^{(+)}_{\vec{k},\vec{k}'}\frac{\mathcal{E}^{*}_{\vec{k}'}}{\sqrt{\mathcal{E}^{*2}_{\vec{k}'} + \Delta^2_{\vec{k}'}}}, \tag{17}$$

where the kernel has the form

$$\mathcal{R}^{(\pm)}_{\vec{k},\vec{k}'} = \frac{|g_{\vec{q}}|^2}{2}\left\{\frac{2\hbar\omega_{\vec{q}}}{(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'})^2 - (\hbar\omega_{\vec{q}})^2} \pm \frac{2\hbar\omega_{\vec{q}}}{(\varepsilon_{\vec{k}} + \varepsilon_{\vec{k}'})^2 - (\hbar\omega_{\vec{q}})^2}\right\}. \tag{18}$$

In these equations, $g_{\vec{q}}$ is the constant of electron–phonon interaction, $\omega_{\vec{q}}$ is the frequency of the phonon characterized by the momentum $\vec{q}$, $\mathcal{E}_k = \hbar^2k^2/2m - \mu$ is the free-electron excitation energy, counted from the level of the chemical potential μ, and $\vec{k}' = \vec{k} - \vec{q}$. As can be easily understood, the kernel $\mathcal{R}^{(\pm)}_{\vec{k},\vec{k}'}$ in equations (16) and (17) describes the contribution of phonon dispersion to the effective electron–electron interactions, leading to formation of a superconducting state. It should be noted once more that no assumption of smallness of the electron–phonon coupling was used in the derivation of the self-consistent equations (16), (17). Thus, since equations (16) and (17) were derived without making any assumption about the value of the constant of the electron–phonon interaction and taking into account the phonon dispersion, these equations can be considered as equations for strong-coupling superconductivity. It this sense, these equations are the direct analogues of Eliashberg equations. In the case of strong coupling, the energy spectrum of the single-particle excitations depends on the form of the phonon spectrum $\omega_{\vec{q}}$. The maximum transmitted wavevector q_0 (phonon momentum) is much smaller than k_F. Therefore, the phonon dispersion relation can be expanded in a series. Taking into account the smallness of q_0, the phonon dispersion can be written in a linear approximation $\omega_{ph} = v_s q$ (v_s is the sound velocity). The accuracy of this approximation is $(v_s/v_F)q_0/k_F \ll 1$.

Equations (16), (17) can be solved approximately—for example, by the iteration method using the BCS gap parameter and the undressed electron spectrum as the zero step. The solution of these equations allows us even at the first step (of order $(q_0/k_F)^2$) to obtain the dependence of the energy gap parameter on the momentum. Then, one can easily analyse the energy spectrum of single-particle excitations. Moreover, the proposed approach takes into account the two aspects of the electron–phonon interaction. The first of these is the direct influence of the interaction on the electron energy in the normal state (as is well known, the electron–phonon interaction leads to the appearance of anomalies in the spectrum of electron excitations in the vicinity of the Fermi level). The second aspect is the Cooper pairing effect, which leads to the superconducting state. As a result, the solution of the self-consistent equations has the form

$$\varepsilon_{\vec{k}} = \sqrt{\tilde{\mathcal{E}}_k^{*2} + \tilde{\Delta}_k^2}, \tag{19}$$

where

$$\tilde{\mathcal{E}}_k^*(k,T) = \mathcal{E}_k(k,T)\frac{g^2}{2\pi^2\hbar}\frac{(2q_0/\tilde{E}_k + F(k))}{1-(g^2/2\pi^2\hbar)F(k)}, \tag{20}$$

$$\tilde{\Delta}_k(k,T) = \Delta_0(T)\frac{(g^2/2\pi^2\hbar)F(k)}{1-(g^2/2\pi^2\hbar)F(k)}. \tag{21}$$

The correction function

$$F(k) = \frac{1}{\hbar v_s}\ln\left|\frac{2\tilde{E}_k - \hbar v_s q_0}{2\tilde{E}_k + \hbar v_s q_0}\right| \tag{22}$$

was introduced in equations (20) and (21). In equation (21), q_0 is the maximum wavevector of the phonon, $\tilde{E}_k$ is the energy spectrum in the weak-coupling limit, and g is the coupling constant. It should be noted that the electron energy correction, the function

$$P(k,T) = \frac{g^2}{2\pi^2\hbar}\frac{(2q_0/\tilde{E}_k + F(k))}{1-(g^2/2\pi^2\hbar)F(k)},$$

is caused by electron–phonon interaction in the normal state. The electron energy is reckoned from the Fermi energy $\mathcal{E}_F$, or more precisely from chemical potential (μ) level. Taking into consideration the expression obtained for the energy excitation spectrum, the linear response function $\alpha(\omega,T)$ can be written as

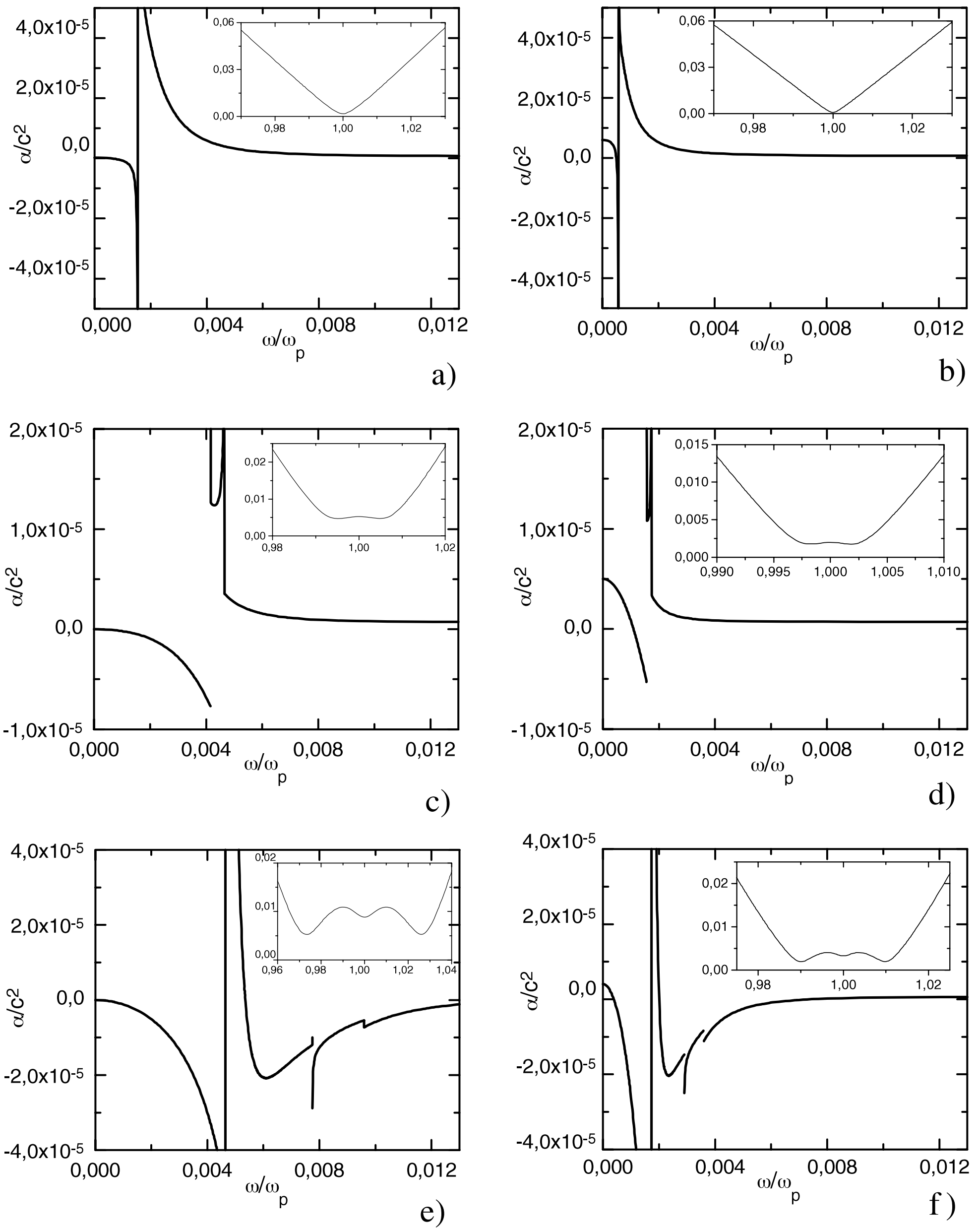

Figure 1. The behaviour of the linear response function $\alpha(\omega, T)$ in the cases of low ($T \approx 0$ K; (*a*), (*c*), (*e*)) and high ($T \approx T_C$; (*b*), (*d*), (*f*)) temperatures. The cases (*a*), (*b*) correspond to weak coupling ($\gamma = 0.2$); the cases (*c*), (*d*) correspond to intermediate coupling ($\gamma = 1.0$); the cases (*e*), (*f*) correspond to strong coupling ($\gamma = 5.0$). The behaviour of the single-particle excitation energy of the superconductor in the vicinity of k_F is shown in the insets.

$$\alpha(\omega, T) = \frac{2\hbar^4(\hbar\omega)^2}{5nm^3(2\pi)^2}\int_0^\infty \frac{k^6\,\mathrm{d}k}{\varepsilon_k^2}\left[\frac{1-2f(\varepsilon_k)}{((\hbar\omega)^2-4\varepsilon_k^2)\varepsilon_k}\left(\tilde{\Delta}_k\frac{\partial\tilde{\mathcal{E}}_k^*}{\partial E_k}-\tilde{\mathcal{E}}_k^*\frac{\partial\tilde{\Delta}_k}{\partial E_k}\right)^2 + \frac{2f(\varepsilon_k)(1-f(\varepsilon_k))}{k_BT(\hbar\omega)^2}\left(\tilde{\Delta}_k\frac{\partial\tilde{\Delta}_k}{\partial E_k}+\tilde{\mathcal{E}}_k^*\frac{\partial\tilde{\mathcal{E}}_k^*}{\partial E_k}\right)^2\right]. \tag{23}$$

The behaviour of the linear response function $\alpha(\omega, T)$ at different temperatures in the case of weak coupling ($\gamma = 0.2$) is shown in figures 1(*a*), (*b*), that for intermediate coupling ($\gamma = 1$) is shown in figures 1(*c*), (*d*), and that for strong coupling ($\gamma = 5$) is shown in figures 1(*e*), (*f*) (the numerical parameter γ characterizes the electron–phonon coupling power). The anomalies of the response function in the strong-coupling case can be explained by extrema in the energy spectrum at frequencies lying in the vicinity of the energy gap. Indeed, in the case of weak coupling, when the bottom of the spectrum of single-fermion excitations has a simple structure (single extremum), the response $\alpha(\omega, T)$ has one singular point. This singularity is located at the frequency corresponding to the superconducting energy gap (see the insets in figure 1). In the case of strong coupling, the bottom of the single-particle excitation spectrum has a complex structure (two additional extrema characterized by the energies above the main energy gap—see figure 1). These additional extrema lead to the additional singularities in $\alpha(\omega, T)$. The space-time Fourier transformation of the conductivity tensor completely defines the behaviour of the dispersion curves of SEW.

3. The dispersion relation for a SEW propagating along a superconductor–vacuum interface

In this section special attention will be paid to propagation of a SEW along a superconductor–vacuum plane interface. The superconductor may be regarded as homogeneous and isotropic, and the degree of electron–phonon coupling is not limited. The Cartesian coordinate system is chosen in such a way that the X- and Y-axes lie on the surface of the crystal and the Z-axis is directed such that the superconductor occupies the half-space $Z > 0$. The SEW propagates along the positive X-axis direction; that is, the wavevector q is directed along the OX-axis.

In the usual fashion we will consider the problem of the excitation of surface waves in the framework of the so-called dielectric formalism [31]. That is, we will use the bulk characteristics for the media (the linear response calculated in the previous section of this work), and the interface will be taken into account by using boundary conditions. Let us assume the following.

(i) The superconductor occupies the whole half-space $Z > 0$. At any rate, the thickness of the superconductor in the direction of the Z-axis is much more than the localization depth, so one can neglect the influence of the bottom interface. The linear dimensions of the superconducting sample in the directions of the X- and Y-axes are much greater than the wavelength of the SEW.

(ii) The dispersion relation is obtained within the framework of the well-known SCIB model [14], where one assumes that the electrons are specularly scattered by the interface and quantum interference effects between the incoming and reflected parts of the electron wavefunction are neglected.

(iii) The influence of the properties of the medium can be taken into consideration within the framework of the Cooper-pair 'jellium' model considered in the modified Bogoliubov method. Taking into account the fact that the penetration length of the SEW is much greater than the interatomic distance, and that the boundary layer is a few lattice constants thick, it is possible to neglect the effects caused by the boundary layer. This means that one can assume a perfect interface to study the SEW. Therefore the bulk characteristics of the medium govern the properties of the SEW.

(iv) The finite 'coherence' length in the superconducting state, which stems from the finite size of the pair bound state, *a priori* necessitates treatment of the surface wave propagation by means of a non-local formalism.

Taking these assumptions into account, in accordance with [14–16], the dispersion relation for the SEW $q_\parallel = q_\parallel(\omega)$ can be represented as the following expression:

$$\left(\frac{\omega}{c}\right)^2 \varepsilon_T(\omega) \pm 2q_\parallel^3 \sqrt{\frac{3\alpha(\omega, T)}{\omega^2}} - q_\parallel^2 \left[\frac{3\alpha(\omega, T)}{c^2} + \varepsilon_T(\omega) + 1\right] = 0 \tag{24}$$

where $q_\parallel$ is the parallel component of the SEW wavevector $\vec{q} = (q_\parallel, 0, q_\perp)$ and $\varepsilon_T = 1 - \omega_p^2/\omega^2$ is the transverse dielectric function. As can be easily seen from equation (9), the characteristic behaviour of the SEW, for both weak and strong coupling, is governed by the $3\alpha(\omega, T)$ factor only.

When analysing the dispersion equation

$$\Omega(q, \omega) = 0 \tag{25}$$

it needs to be taken into account that there are two types of wave propagation in the system with dispersed parameters [31, 33]: that with initial conditions and that with boundary conditions. In the first case the state of the system at a certain moment is determined by an evaluation equation in accordance with its initial state. In the other case the state of the system at any point of the space is determined by an evaluation equation taking into consideration the state of the system at the boundary (the system is acted on by external influences). In the first case the solution of the dispersion equation takes the form

$$\omega = \omega(q). \tag{26}$$

In the second case the solution of the dispersion equation is sought in the form

$$q = q(\omega). \tag{27}$$

It should be noted that different solutions (equations (26) and (27)) of the problem are connected with essentially different experimental set-ups.

Dispersion equation (24) is written as a polynomial in $q_\parallel$ and its solution can be represented in the form $q_\parallel = q_\parallel(\omega)$. This form of dispersion equation corresponds to the problem with boundary conditions. It is evident that equation (24) has six solutions which are in matched pairs with opposite signs. At the same time they are limited by the condition $\mathrm{Re}[q_\parallel]\,\mathrm{Im}[q_\parallel] > 0$. Then, solving equation (24) one can automatically obtain the dispersion characteristics for SEW which are propagating along the positive and negative X-axis directions. Generally speaking, equation (24) can be solved by numerical methods. However, one can derive some important conclusions without using numerical calculations.

First of all, one needs to determine the character and number of solutions of equation (24) in different frequency ranges. It is clear that the analysis of equation (24) significantly depends on the sign of the linear response function. In the case where $\alpha > 0$, equation (24) is a cubic equation with real coefficients and can be presented as

$$\pm 2\sqrt{\eta}\, q_\parallel^3 - (\eta + \varepsilon_T + 1) q_\parallel^2 k_0 + k_0^3 \varepsilon_T = 0, \tag{28}$$

where $\eta = |3\alpha/c^2|$, $k_0 = \omega/c$.

The analysis of its roots can be approached by studying the relation between the signs of the cubic parabola at the extremum points, which are determined as

$$\begin{aligned} q_{\parallel_{01}} &= 0, & f_1 &= F(q_{\parallel_{01}}) = k_0^3 \varepsilon_T, \\ q_{\parallel_{02}} &= \pm\frac{\eta + \varepsilon_T + 1}{3\sqrt{\eta}}, & f_2 &= F(q_{\parallel_{02}}) = k_0^3 \left[\left(\frac{\eta + \varepsilon_T + 1}{3}\right)^3 \frac{1}{\eta} + \varepsilon_T\right]. \end{aligned} \tag{29}$$

All possible solutions of equation (24) in the case where $\alpha > 0$ are shown in table 1.

Table 1. The types of behaviour exhibited by the solutions of the dispersion equation (equation (24)) in the range $\alpha > 0$.

f_1	f_2	Condition	Solution
0	>0	$\varepsilon_T = 0$	$q_{\parallel 1} = q_{\parallel 2} = 0$, two degenerate branches
0	<0		$q_{\parallel 3} \neq 0$, dissipationless branch
>0	0	$\left(\frac{\eta + \varepsilon_T + 1}{3}\right)\frac{1}{\eta} + \varepsilon_T = 0$	$q_{\parallel 1} = q_{\parallel 2} \neq 0$, two dissipationless and degenerate branches
<0	0		$q_{\parallel 3} \neq q_{\parallel 1,2} \neq 0$, three dissipationless branches
>0	<0	$\left[\left(\frac{\eta + \varepsilon_T + 1}{3}\right)\frac{1}{\eta} + \varepsilon_T\right]\varepsilon_T < 0$	$q_{\parallel 1} \neq q_{\parallel 2} \neq q_{\parallel 3}$, three dissipationless branches
<0	>0		
>0	>0	$\left[\left(\frac{\eta + \varepsilon_T + 1}{3}\right)\frac{1}{\eta} + \varepsilon_T\right]\varepsilon_T > 0$	$q_{\parallel 1}$, dissipationless branch
<0	<0		$q_{\parallel 2} = q^*_{\parallel 3}$, two dissipative branches, one of them unstable

Table 2. The types of behaviour exhibited by the solutions of the dispersion equation (equation (24)) in the range $\alpha < 0$.

f_1	f_2	Condition	Solution
0	>0	$\varepsilon_T = 0$	$q_{\parallel 1} = q_{\parallel 2} = 0$ are degenerate branches
0	<0		$q_{\parallel 3} \neq 0$ is a radiation branch
>0	0	$\left(\frac{-\eta + \varepsilon_T + 1}{3}\right)\frac{1}{\eta} + \varepsilon_T = 0$	$q_{\parallel 1} = q_{\parallel 2} \neq 0,\ q_{\parallel 3} \neq q_{\parallel 1,2} \neq 0$
<0	0		are three radiation branches
>0	<0	$\left[\left(\frac{-\eta + \varepsilon_T + 1}{3}\right)\frac{1}{\eta} + \varepsilon_T\right]\varepsilon_T < 0$	$q_{\parallel 1} \neq q_{\parallel 2} \neq q_{\parallel 3}$
<0	>0		are three radiation branches
>0	>0	$\left[\left(\frac{-\eta + \varepsilon_T + 1}{3}\right)\frac{1}{\eta} + \varepsilon_T\right]\varepsilon_T > 0$	$q_{\parallel 1}$ is a radiation branch
<0	<0		$q_{\parallel 2} = q^*_{\parallel 3}$ are two dissipative branches, one of them unstable

Similar analysis of equation (24) when $\alpha < 0$ can be carried out by making the replacement $q_\parallel = \mathrm{i}q'_\parallel$, because equation (24) transforms into a cubic equation with real coefficients:

$$\pm 2\sqrt{\eta}{q'_\parallel}^3 + (-\eta + \varepsilon_T + 1)k_0 {q'_\parallel}^2 + k_0^3 \varepsilon_T = 0. \tag{30}$$

The coordinates of the extrema of the cubic parabola in this case are

$$\begin{aligned} q'_{\parallel 01} &= 0, & f_1 &= F(q'_{\parallel 01}) = k_0^3 \varepsilon_T, \\ q'_{\parallel 02} &= \mp \frac{-\eta + \varepsilon_T + 1}{3\sqrt{\eta}} k_0, & f_2 &= F(q'_{\parallel 02}) = \left[\left(\frac{-\eta + \varepsilon_T + 1}{3}\right)^3 \frac{1}{T} + \varepsilon_T\right] k_0. \end{aligned} \tag{31}$$

All possible solutions of equation (24) in the case where $\alpha < 0$ are shown in table 2.

To analyse the dispersion curves one can make some simplifications. One notes that the inequality $3\alpha/c^2 \ll \varepsilon_T$ is fulfilled over the whole frequency range except at some points where $3\alpha/c^2 \to \pm\infty$. This makes the numerical analysis rather difficult.

In the case of large values of $q_\parallel$ one can omit the term $(\omega/c)^2\varepsilon_T(\omega)$ in equation (24). Then the dispersion relation becomes

$$\pm 2 q_\parallel^3 \sqrt{\frac{3\alpha(\omega, T)}{\omega^2}} - q_\parallel^2 \left[\frac{3\alpha(\omega, T)}{c^2} + \varepsilon_T(\omega) + 1\right] = 0. \tag{32}$$

This equation has two solutions:

$$q_\parallel = \pm \frac{1}{2} \frac{[3\alpha(\omega, T)/c^2 + \varepsilon_T(\omega) + 1]}{\sqrt{3\alpha(\omega, T)/\omega^2}}. \tag{33}$$

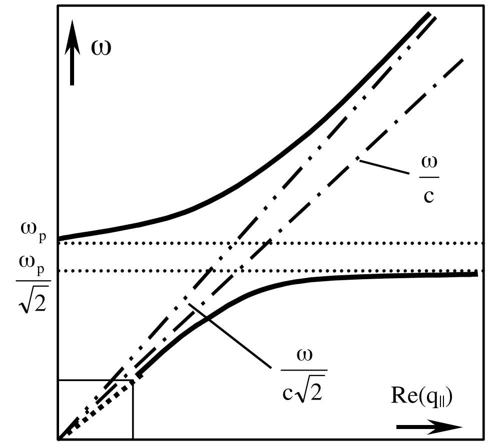

Figure 2. The dispersion curves of SEW in the long-range limit.

These solutions are proportional to $\frac{1}{2}(\omega/c)\sqrt{3\alpha(\omega, T)}$ at rather large values of the response function $\alpha(\omega, T)$; i.e. at $\alpha \to \infty$ one has $q_\parallel \to \infty$.

A more interesting case is realized in the region of small wavevectors. Analytical consideration can be performed in the local regime when $q_\parallel \ll k_F$. As a result, equation (24) can be rewritten in the form

$$q_\parallel = \frac{\omega}{c}\left[\frac{\varepsilon_T(\omega)}{3\alpha(\omega, T)/c^2 + \varepsilon_T(\omega) + 1}\right]^{1/2}. \tag{34}$$

The expression in brackets is positive when $\varepsilon_T(\omega)$ and $[3\alpha(\omega, T)/c^2 + \varepsilon_T(\omega) + 1]$ are of the same sign. This case corresponds to propagation of the wave without energy dissipation. In the frequency range $\omega < \omega_p$ the dielectric function is negative, $\varepsilon_T(\omega) < 0$; then the propagation of the SEW happens without energy dissipation. This SEW can be observed in the frequency range where the inequality $3\alpha(\omega, T)/c^2 < -(1 + \varepsilon_T(\omega))$ is fulfilled. Indeed, since the expression $|3\alpha(\omega, T)/c^2|$ is much less than $|1 + \varepsilon_T(\omega)|$ and is positive everywhere except in some narrow neighbourhoods between the frequencies ω_{Ij} and ω_{Zj}, the wavevector $q_\parallel$ will be real everywhere except in narrow ranges in the vicinity of resonances of the linear response function, where dissipation (which is proportional to $\mathrm{Im}\{q_\parallel\}$) drastically increases. Taking it into account that $3\alpha(\omega \gg \omega_g)/c^2 \ll \varepsilon_T$ (when $\omega_g = 2\tilde{\Delta}(k_F, T)/\hbar$), one can see that the dispersion relation turn into the classic form [10]

$$q_\parallel = \frac{\omega}{c}\left(\frac{\varepsilon_T}{1 + \varepsilon_T}\right)^{1/2}.$$

Then, the dispersion curves in the long-wavelength limit have the form shown in figure 2.

To obtain a clearer understanding of the results of numerical calculations, let us analyse the behaviour of the dispersion of the SEW (at $q_\parallel \ll k_F$) using equation (34). A sketch of the behaviour of the dispersion curves at the singular point ω_Z is shown in figure 3. The behaviour of the normalized linear response function $3\alpha/c^2$ is shown in figure 3(a) as solid curve. One can see that $\alpha \to -\infty$ at $\omega \to \omega_Z - 0$, and the denominator $3\alpha/c^2 + 1 + \varepsilon_T$ in equation (34) becomes infinity large. Then the dispersion equation has a solution at $q_\parallel \to 0$. At $\omega \to \omega_Z + 0$ the linear response function $\alpha \to +\infty$; then there exists a point ω_i at which the equality

$$\frac{3\alpha(\omega)}{c^2} = -[1 + \varepsilon_T(\omega)] \tag{35}$$

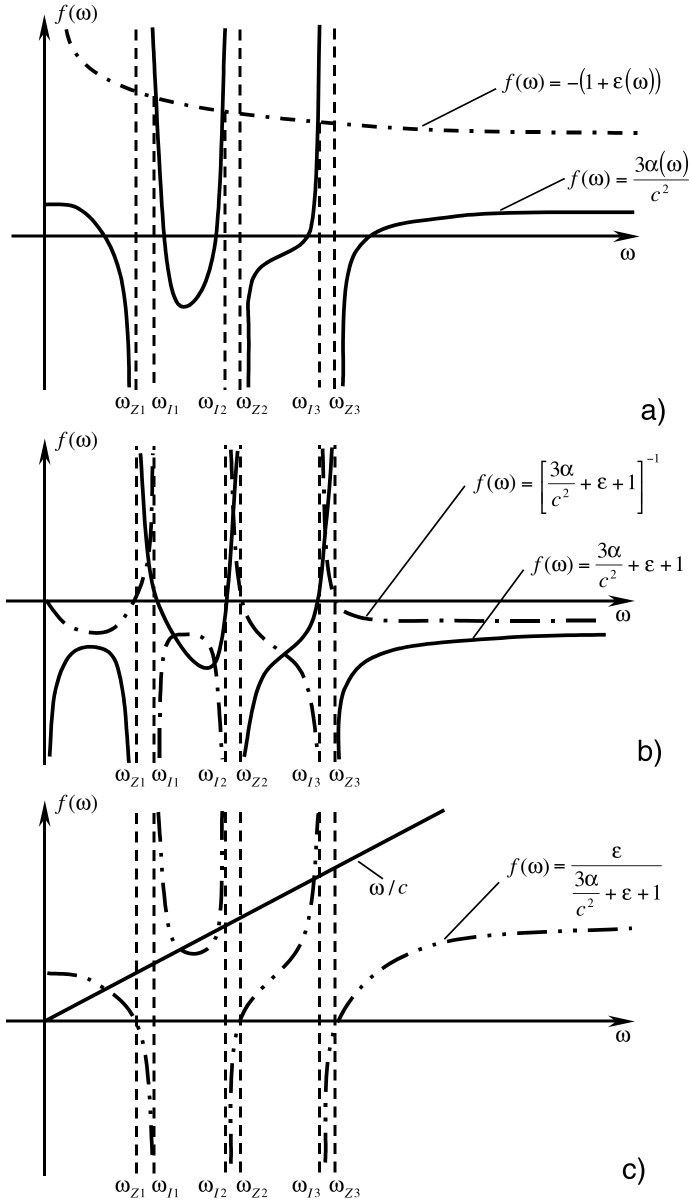

Figure 3. Sketches of the graphical analysis of the dispersion curves.

holds, i.e. the denominator on the right in equation (32) becomes equal to zero. As a result, the line $\omega = \omega_i$ becomes asymptotic to the dispersion curve as $q_\parallel \to \infty$. The behaviour of the right-hand side of equation (35) is shown in figure 6(*a*) by a dash–dot curve. The behaviours of $3\alpha(\omega)/c^2 + 1 + \varepsilon_T(\omega)$ and $[3\alpha(\omega)/c^2 + 1 + \varepsilon_T(\omega)]^{-1}$ are shown by solid and dash–dot curves, respectively, in figure 3(*b*). In the gap between ω_Z and ω_i the inequality $[3\alpha/c^2 + 1 + \varepsilon_T]^{-1} < 0$ is fulfilled. Then, the condition for existence of the SEW, $\mathrm{Re}[q_\parallel]\,\mathrm{Im}[q_\parallel] > 0$ is not satisfied. This means that the dispersion of the SEW must have an energy gap in the range $\omega_Z < \omega < \omega_i$.

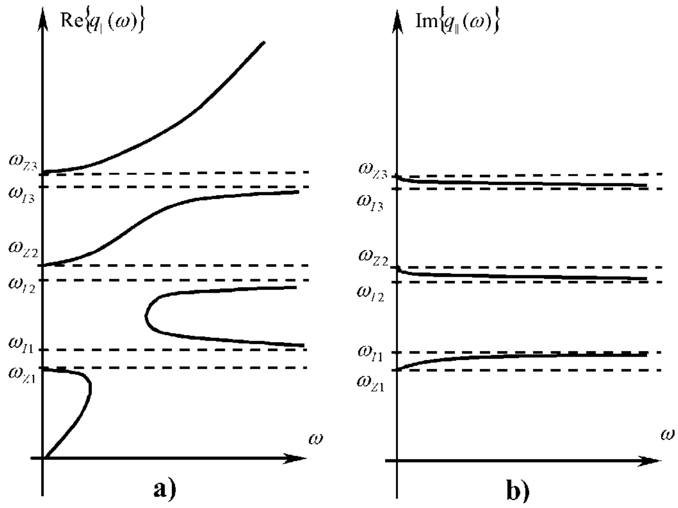

Figure 4. Sketches of the behaviour of the real (a) and imaginary (b) parts of the wavevector $q_\parallel$ of the SEW (in the case of strong coupling and $T \approx 0$ K).

The dependence of the factor $\varepsilon_T[3\alpha/c^2 + 1 + \varepsilon_T]$ on the frequency and the line $k = \omega/c$ are given in figure 3(c) as dash–double-dot and solid curves, respectively. The solid curves show the SEW dispersion in the frequency domain under consideration. Schematic diagrams of the real and imaginary parts of $q_\parallel$ are presented in figures 4(a) and (b), respectively. Analogously, the pictures for weak, intermediate, and strong coupling for high and low temperatures are presented in figures 5(a)–(f), for the real part, and in figures 6(a)–(f), for the imaginary part of $q_\parallel$.

4. Discussion

Considering the non-local electrodynamical problem of surface electromagnetic wave propagation in a superconductor, an attempt to go beyond the weak-coupling approximation is made in the present work. The modified u–v transformation method was used for this purpose. It was shown that the modification of the dispersion relation is connected with the presence of addition anomalies in the frequency dependence of the linear response function. The main transformation of the linear response function is connected with the change of the quasi-particle excitation spectrum in the vicinity of the Fermi momentum.

Numerical analysis showed that the main transformation in a superconductor spectrum is connected with the blurring of the BCS dependence of the energy gap on the wavevector close to the Fermi wavevector. The domain of blurring has a size similar to the magnitude of the maximum wavevector of the virtual phonon, as corroborated in the work [14]. The additional extrema in the single-fermion spectrum in the superconducting state for strong coupling bring into existence additional (in comparison to the case of a weak-coupling superconductor) branches of the SEW dispersion law. The dispersion of the SEW, for both weak- and strong-coupling superconductors, is shown in figure 7 for different temperatures, $T = 0.1$ and 80 K. It should be pointed that the behaviours of the SEW dispersion in these cases are essentially different. Indeed, as was mentioned above, due to additional extrema in the spectrum of single-fermion excitations of the strong-coupling superconductor the susceptibility $\alpha(\omega, T)$ is characterized by additional anomalies. These anomalies occur at the frequencies

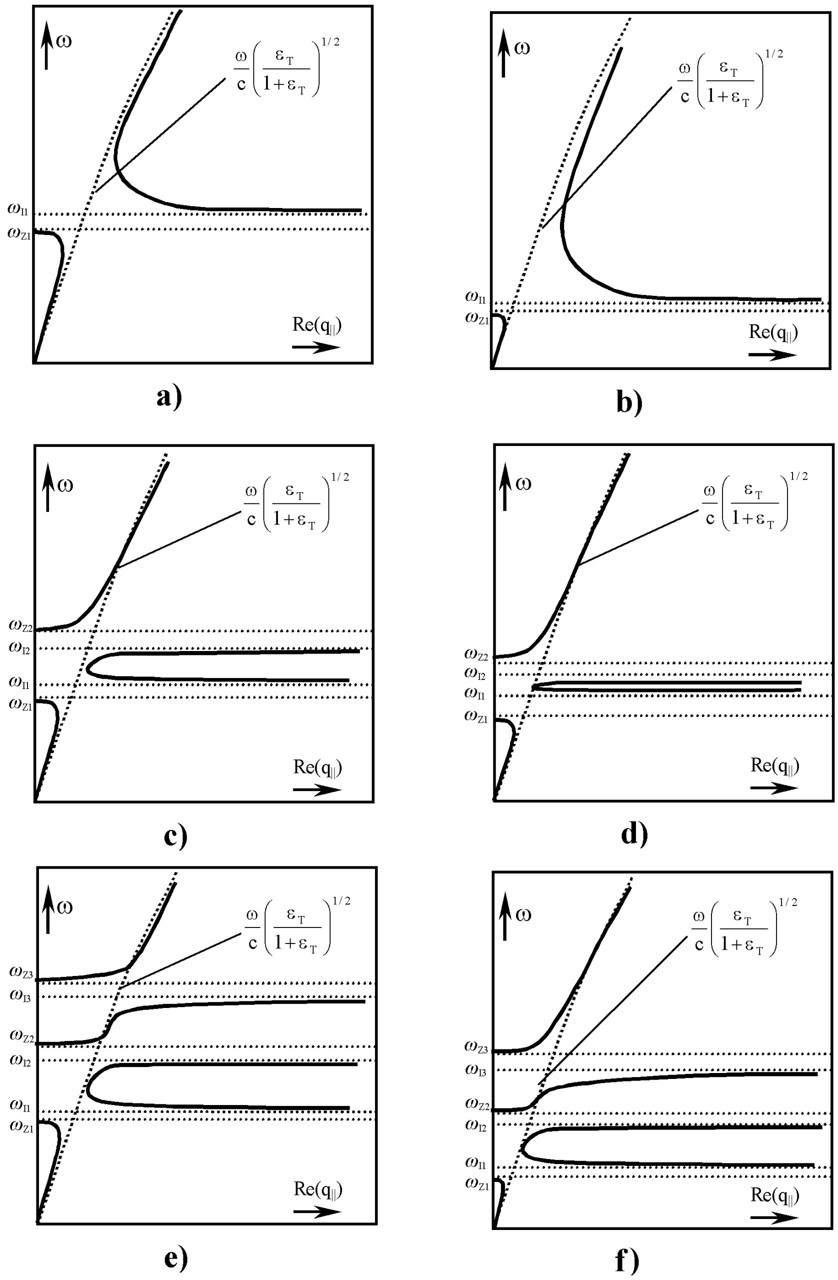

Figure 5. The real part of the wavevector $q_{\|}$ of the SEW in cases of low ($T \approx 0$ K; (*a*), (*c*), (*e*)) and high ($T \approx T_C$; (*b*), (*d*), (*f*)) temperatures. The cases (*a*), (*b*) correspond to weak coupling ($\gamma = 0.2$); the cases (*c*), (*d*) correspond to intermediate coupling ($\gamma = 1.0$); the cases (*e*), (*f*) correspond to strong coupling ($\gamma = 5.0$).

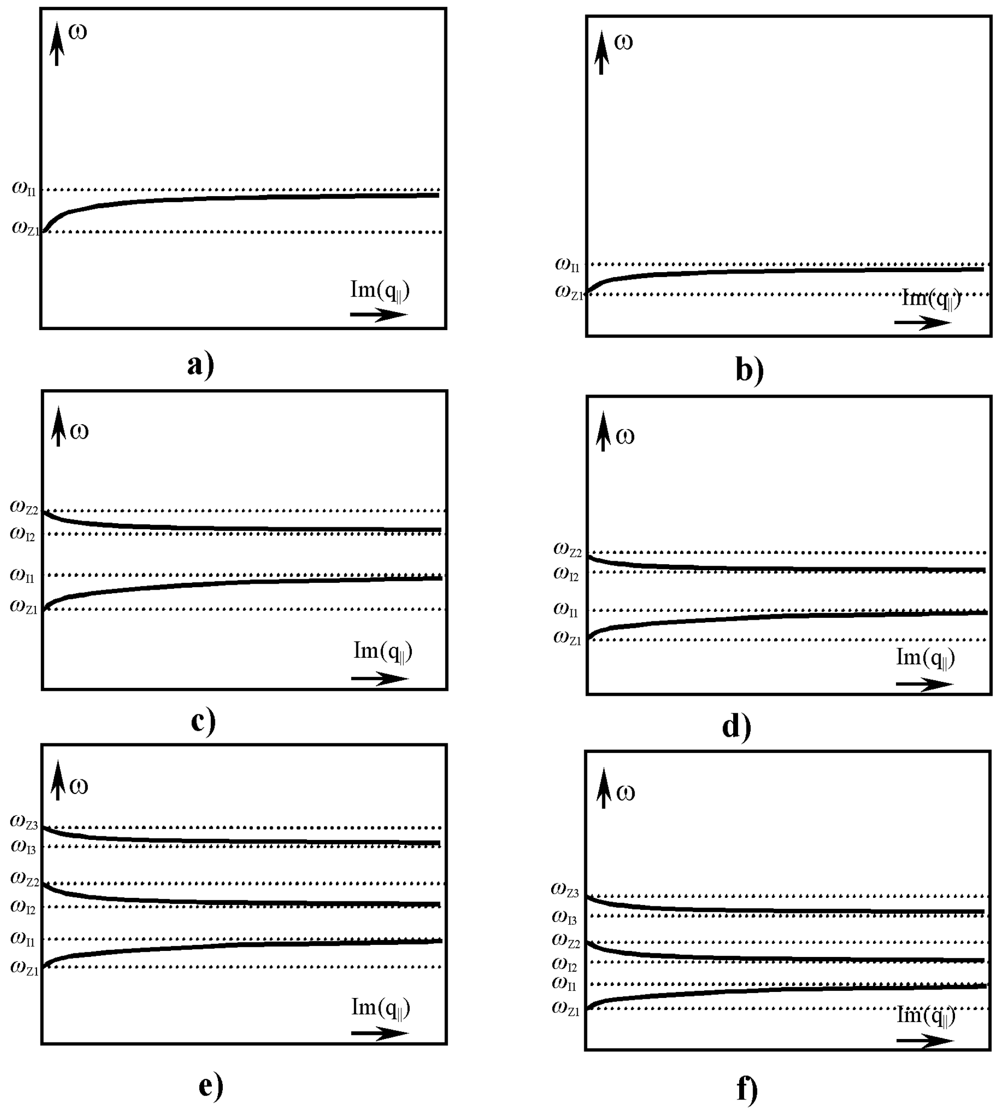

Figure 6. The imaginary part of the wavevector $q_{\|}$ of the SEW in cases of low ($T \approx 0$ K; (*a*), (*c*), (*e*)) and high ($T \approx T_C$; (*b*), (*d*), (*f*)) temperatures. The cases (*a*), (*b*) correspond to weak coupling ($\gamma = 0.2$); the cases (*c*), (*d*) correspond to intermediate coupling ($\gamma = 1.0$); the cases (*e*), (*f*) correspond to strong coupling ($\gamma = 5.0$).

corresponding to extrema of the single-fermion excitation spectrum. The anomalies of $\alpha(\omega, T)$ can be connected with additional volume modes of oscillations of the electron plasma in the superconducting state. These plasma modes interact with the oscillations of the exciting electromagnetic field. As a result, in the domains of phase synchronization the dispersion curves split and the energy gaps in the SEW dispersion law appear. There are three gaps: $\Delta_{1S} = \omega_{i1} - \omega_{Z1}$, $\Delta_{2S} = \omega_{Z2} - \omega_{i2}$, and $\Delta_{3S} = \omega_{Z3} - \omega_{i3}$. One should note that, according to general principles of analysis of the stability of the excitations (see, for example [32, 33]), the SEW in the frequency–momentum domain near the energy gap Δ_{2S} can be unstable.

The qualitative analysis by Sturrock [32] gives grounds for suggesting that this is convective instability.

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